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Moving From a Sketch to a Painting: Toward an Informative Mobility Effect

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Supplementary Materials: Materials [see Index of Supplementary Materials]

Abstract

This article develops an identification strategy for estimating the mobility effect in linear models by enriching the mobility term with theoretically grounded proxy measures that do not linearly covary with class origin and destination. It introduces three substantively informative metrics of social mobility: permeability, defined as the difference in frequency or probability mass between mobility positions and their corresponding nonmobile diagonals; atypicality, captured by the deviation between individuals' predicted probabilities of occupying a given mobility position and those associated with the diagonal; and social distance, measured as the covariate-based Gower distance between mobility positions and their diagonal counterparts. By incorporating these proxies, the proposed framework enables simultaneous estimation of the net effects of origin, destination, and mobility, while relaxing restrictive functional constraints in conventional specifications. This reformulation effectively addresses previously underexplored research scenarios, including reference-group shifts, counterfactual decomposition, sociodemographic matching, and continuous measures of socioeconomic status. Two empirical examples are provided to demonstrate the analytical leverage and interpretive gains of the proposed approach.

Keywords

diagonal reference model, measure of mobility, Gower distance, social mobility

The diagonal reference model (DRM) (Sobel, 1981, 1985) has long been serving as the workhorse for revealing the consequences of social mobility, including but not limited to fertility, health, and self-rated social status (for a review, see Zang et al., 2023). By configuring the immobility term as a weighted sum of relevant estimated cell means along the diagonal of a mobility table, DRM enables scholars to estimate the "net" mobility effect from origin i to destination j, relative to the nonmobile reference composed of the diagonal cells (i,i) and (j,j).

Despite its widespread use, DRM has been subject to sustained criticism. First, because the immobility reference is specified as a weighted average of diagonal cell means, the effects of class origin and class destination — similar to the identification structure in Hope's diamond model (Hope, 1975) — are not directly estimated or independently controlled for within the DRM specification. As a result, the estimated mobility effect is taken to be "net" not in the sense that it is *additive* to the independent origin and destination effects, but rather because it is relative to a composite of the latter two. This configuration can be unsatisfactory, especially when the analytical goal is to disentangle the *independent* contributions of origin, destination, and mobility to a given outcome.

Second, a prevalent analytical strategy in sociodemographic research is counterfactual decomposition, wherein selected components of a statistic are altered while the other components are held constant to see how this statistic varies in an imagined counterfactual scenario (i.e., Breen & Andersen, 2012). To date, however, it remains unclear how to incorporate the counterfactual thinking into the DRM framework, i.e., how to estimate the mobility effect for a cohort under a counterfactual diagonal reference of a different cohort.

Third, the theoretical rationale of DRM hinges on the assumption that diagonal cells best capture the persistent and stable characteristics of some particular social status insofar as to serve as the immobility reference. Yet, this assumption may not hold, or be impractical, in certain contexts. In rapidly transitioning societies, for instance, mobility patterns are often driven by broad structural forces, thereby weakening the reference value of the diagonal cells (Luo, 2022). Similar challenge arises with income mobility: the continuous nature of income precludes the construction of meaningful "diagonal cells" (Hu & Zhou, 2024). These cases underscore the need for further extensions of DRM.

In this article, we propose a diagonal reference model using proxy variables (DRM-PV) in response to these extension needs. Specifically, we shift the focus from the construction of *immobility* in the conventional DRM to a proxy-based enrichment of the *mobility* term that does not linearly covary with origin and destination. DRM-PV enables: (1) estimation of the mobility effect that is net of independent origin and destination effects, (2) analysis under counterfactual diagonal settings, (3) flexible specification of the reference, and (4) application to continuous measures of socioeconomic status such as income. Note that DRM-PV, by construction, resembles DRM as it retains the linear model structure to capture within-person, individual-level mobility effects, which diverges from recent efforts to address the identification problem by reframing the mobility effect as a conditional causal effect — whether at the individual level (Breen & Ermisch, 2024) or the societal level (Wei & Xie, 2025). However, due to the different identification strategies, DRM-PV stands for a different model relative to the conventional DRM.

The remaining parts of this article are organized as follows: The second section reviews the identification problem, existing strategies, and importantly, the rationale of DRM. The third section presents the technical details of DRM-PV and addresses the methodological advantages of DRM-PV. The fourth section presents two empirical illustrative examples. We conclude this study in the fifth section.

The Identification Problem and Existing Handling Approaches

The Identification Problem

When attempting to simultaneously estimate the effects of origin, destination, and mobility, the model identification problem arises when mobility is measured as the difference between origin and destination (Hu & Zhou, 2024). Formally, one may fit the following linear model:

$$Y_{ijk} = \beta_0 + \beta_1 O_{ijk} + \beta_2 D_{ijk} + \beta_3 \left(O_{ijk} - D_{ijk} \right) + \varepsilon_{ijk}, \tag{1}$$

where Y_{ijk}, O_{ijk}, and D_{ijk} are respectively outcome, origin, and destination variables for unit k at origin i and destination j, and ε_{ijk} stands for the random error. Model (1) is clearly unidentifiable due to the perfect multicollinearity between O_{ijk}, D_{ijk}, and $O_{ijk} - D_{ijk}$.

The essential reason for this identification problem lies in the fact that the measure of mobility is constructed as a linear combination of origin and destination. Consequently, the mobility term contributes no additional information, so it is impossible to extract an independent estimate for its effect. Another way to understand this problem is to recognize that, for a given origin-destination pair (i, j), the mobility term degenerates to a fixed value j-i, leaving no room to examine its *co-variation* with Y.

How to Address the Identification Problem: An Overview

The identification problem encountered in estimating mobility effects also arises in other research domains, most notably in age–period–cohort (APC) analysis (Fosse & Winship, 2019; O’Brien, 2014). Drawing on the APC literature, three general identification strategies can be employed to obtain point, rather than merely bounded, estimates of the mobility effect.

The first, and perhaps most intuitive, strategy is to introduce nonlinearity into one or more components of the model — for example, by incorporating quadratic terms (Yang & Land, 2006) or by isolating and modeling nonlinear effects separately (Fosse & Winship, 2018). This approach, however, is defensible only insofar as the imposed or disentangled nonlinear specifications possess clear substantive justification rather than serving merely as technical devices for identification. Absent such justification, introduction of nonlinearity amounts to little more than a *mechanical* workaround to the identification problem.

The second strategy substitutes proxy variables for one or more of the origin, destination, or mobility terms (Glenn, 2005). Rather than relying on generic indicators of these variables, this approach seeks to specify *which substantive dimensions* of, for instance, mobility, are operative in shaping the outcome. Crucially, the proxy must be distinctive to the construct it represents, a requirement that typically demands explicit theoretical justification.

The third strategy relaxes this requirement by adopting a *mechanism*-based approach, in which multiple mediating variables link the linearly covarying predictors (origin, destination, and mobility) to the outcome (Winship & Harding, 2008). By introducing these mediators, the model becomes overidentified, and the estimation of the "mobility effect" is reframed as an investigation of the causal processes through which mobility influences the outcome.

In empirical research on mobility effects, the nonlinearity-based strategy has been most commonly employed. For example, Duncan's (1966) square additive model specifies mobility as the production of origin and destination, and Hope's (1975) diamond model addresses the identification problem by combining origin and destination into a single variable representing a shared dimension of social status. Beyond these more traditional approaches, the strategy that has seen the widest contemporary application is DRM, which is discussed in detail in the following section.

DRM

Relative to the unidentifiable Model (1), DRM retains the mobility term $O_{ijk} - D_{ijk}$, but, like the diamond model, giving up estimating individual origin and destination effects. Specifically, DRM fits the following linear model:

$$Y_{ijk} = w \times \mu_{ii} + (1 - w) \times \mu_{jj} + \beta_1 \left(O_{ijk} - D_{ijk}\right) + \varepsilon_{ijk}, \tag{2}$$

where μ_{ii} is the mean of Y in the diagonal cell (i, i), μ_{jj} represents the mean of Y in the diagonal cell (j, j), w stands for the strength of the effect of origin i relative to that of destination j, which lies in the interval [0, 1]. μ_{ii}, μ_{jj}, and w are estimated from data. Taken together, $w \times \mu_{ii} + (1 - w) \times \mu_{jj}$ represents the immobility term (sometimes called intercept) for each off-diagonal cell in the mobility table, relative to which the mobility effect is estimated. This modeling configuration suggests that DRM circumvents the identification problem by configuring the immobility term $w \times \mu_{ii} + (1 - w) \times \mu_{jj}$ in a way that is not a linear combination of origin O_{ijk} and destination D_{ijk}. Therefore, this term does not linearly vary with $O_{ijk} - D_{ijk}$.

DRM has been widely used to examine the effects of mobility across a broad range of outcomes. This widespread adoption, however, does not imply that DRM is a panacea. As noted earlier, DRM relinquishes the explicit control over the independent origin (O_{ijk}) and destination (D_{ijk}) terms available in Model (1), so it does not permit the separate estimation of origin and destination effects. From a methodological standpoint, DRM assumes that the linear effects of origin and destination are adequately captured by the weighted combination $w \times \mu_{ii} + (1 - w) \times \mu_{jj}$. As a consequence, the model imposes an implicit constraint that, as shown by Fosse and Pfeffer (2025), could mechanically force the linear mobility effect toward zero. On this basis, Fosse and Pfeffer (2025) argues that the mobility effect cannot be estimated using DRM unless the modeling assumptions can be supported.

Essentially, the strategy DRM employs to circumvent the identification problem remains one of nonlinearity — specifically, the construction of an immobility term that does not linearly covary with the mobility term. In effect, this amounts to little more than resolving rank deficiency in the design matrix, rather than addressing the underlying conceptual indeterminacy. In response, the present study adopts a proxy-variable strategy, extending the DRM framework by incorporating more informative measures of the mobility term.

Diagonal Reference Model Using Proxy Variables

Proxy Variables for Mobility

In this article, we address the identification problem in estimating the mobility effect by adopting the proxy-variable strategy. Specifically, we construct new measures of the mobility term that do not covary linearly with origin and destination. Building on theoretical discussions of the processes and lived experiences of social mobility — most notably those articulated by Pitirim Sorokin (1959) — there are multiple directions along which mobility-relevant proxy variables can be specified.

One such proxy is the *permeability* of the destination relative to the origin, defined as the degree of ease with which boundaries between origin and destination positions can be crossed. This measure does not depend on individual-level characteristics; instead, it captures the structural properties of barriers between social positions. A natural empirical manifestation of such accessibility is the observed frequency or probability mass, as it directly reflects the likelihood that individuals occupy a given mobility position. Accordingly, we operationalize the permeability of mobility from origin i to destination j as the absolute value of the difference between the empirical frequency or probability mass of destination j and that of origin i, as denoted by:

$$\left|\widehat{f}_{i..} - \widehat{f}_{.j.}\right|,$$

where the subscript dots stand for the averaging of the corresponding dimensions (i.e., $\widehat{f}_{i..}$ only concerns about origin, so the dimensions of destination j and unit k should be averaged out).

If we further consider how individual characteristics may come into play, two additional proxy variables can be constructed. Based on individual-level covariates, we can predict an individual's probability of occupying the origin position and the probability of occupying the destination position. The difference between these two probabilities captures how rare or atypical a given mobility transition is, conditional on personal attributes. This atypicality-based proxy thus reflects the extent to which a mobility transition deviates from what would be expected given individual characteristics. Formally, it is defined as the absolute value of the difference between the predicted probabilities for individual k to be located in origin i and destination j, denoted by

$$\left|\widehat{p}_{i\,.\,k} - \widehat{p}_{\,.\,jk}\right|.$$

A third strategy draws directly on the multivariate attributes of individuals. This strategy has the merit of reducing dependence on the (usually parametric) model to predict probabilities. Under this approach, mobility is characterized by the distance between the origin i and destination j computed from their multivariate profiles. This distance captures how socially dissimilar the two positions are, given the full configuration of characteristics that define the individuals who occupy them. In this sense, the proxy can be interpreted as representing the social — or broadly "cultural" — distance between origin and destination. Let $\bar{x}$ denote the vector of average d-dimensional multivariate characteristics. The proxy variable is then defined as the absolute value of the difference between the mean attribute vectors of the origin and destination positions, $\left|\bar{x}_{i..} - \bar{x}_{\,.\,j\,.}\right|$, which can be further summed up to stand for the distance metric, that is,

$$\sum_{d} \left|\bar{x}_{i..} - \bar{x}_{\,.\,j\,.}\right|.$$

Larger values indicate greater social distance between origin and destination, and thus a mobility transition that entails more substantial dislocation.

In sum, approaching mobility from multiple perspectives yields three distinct ways of constructing proxy variables for the mobility term, as summarized in Table 1. It is worth noting that our focus here is on the extent of mobility, and the direction of mobility (upward or downward) is not part of the construction of these proxies.

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Table 1

Proxies for Mobility

Proxies	Permeability	Atypicality	Social Distance
Proxy without Diagonal Reference	$\lvert \hat{f}_{i..} - \hat{f}_{.j.} \rvert$	$\lvert \hat{p}_{i.k} - \hat{p}_{.jk} \rvert$	$\sum_d \lvert \bar{x}_{i..} - \bar{x}_{.j.} \rvert$
Proxy with Diagonal Reference	$\frac{\lvert \hat{f}_{ij.} - \hat{f}_{ii.} \rvert + \lvert \hat{f}_{ij.} - \hat{f}_{jj.} \rvert}{2}$	$\frac{\lvert \hat{p}_{ijk} - \hat{p}_{iik} \rvert + \lvert \hat{p}_{ijk} - \hat{p}_{jjk} \rvert}{2}$	$\frac{\sum_{dk} \lvert x_{ijk} - \bar{x}_{ii.} \rvert + \sum_{dk} \lvert x_{ijk} - \bar{x}_{jj.} \rvert}{2}$

Note. $\hat{f}_{i..}$ = the estimated frequency of origin i, $\hat{f}_{.j.}$ = the estimated frequency of destination j, $\hat{f}_{ij.}$ = the estimated frequency of cell i–j, $\hat{f}_{ii.}$ = the estimated frequency of diagonal cell i–i, $\hat{f}_{jj.}$ = the estimated frequency of diagonal cell j–j, $\hat{p}_{i.k}$= the estimated probability for individual k to be located in origin i, $\hat{p}_{.jk}$= the estimated probability for individual k to be located in destination j, $\bar{x}_{i..}$= the average of the d-dimensional multivariate characteristics of origin i, $\bar{x}_{.j.}$ = the average of the d-dimensional multivariate characteristics of destination j, x_{ijk}= the d-dimensional multivariate characteristics of individual k at the cell i–j, $\bar{x}_{ii.}$ = the average of the d-dimensional multivariate characteristics of diagonal cell i–i, $\bar{x}_{jj.}$ = the average of the d-dimensional multivariate characteristics of diagonal cell j–j.

The Necessity of Preserving the Diagonal Reference

The proxy variables for the mobility term are substantively more informative than a simple difference between status indicators. Nevertheless, substituting the original mobility term with these proxies does not, in itself, fully resolve the identification problem. The reason is straightforward: if $O_{ijk} - D_{ijk}$ in Model (1) can be replaced by the difference between two proxy variables — for example, $\hat{f}_{i..} - \hat{f}_{.j.}$ — then the main effects of origin and destination can likewise be substituted with their corresponding proxies. In this case, the model is merely estimated using a different set of variables, while the underlying multicollinearity problem remains intact.

One might alternatively substitute only the mobility term with its proxy while retaining the original indicator measures for origin and destination. However, such an identification strategy is *mechanical* and *conceptually unsatisfactory*. If the proxy variables are indeed more informative and substantively meaningful, there is no principled reason to restrict their use to the mobility term alone. As long as the proxies are capable of representing the mobility term, they are equally capable of proxying origin and destination, and selectively substituting only one term does not well address the core identification issue.

This point highlights a fundamental difference between the estimation of mobility effect and that of APC effects. In APC analysis, age, period, and cohort each corresponds to a substantively distinct dimension of variation in the outcome. Although, from a mathematical standpoint, age can be expressed as the difference between survey period and birth cohort, its substantive meaning is *not derived from that difference*. Age is theoretically defined in its own right. By contrast, mobility is substantively defined as a difference between origin and destination. Consequently, if mobility can be represented by differences in theoretically grounded proxy variables, there is a corresponding substantive rationale for proxying class origin and class destination in a parallel manner. This definitional interdependence underlies why identification strategies that merely substitute the mobility term — while preserving the original indicators for origin and destination — are conceptually fragile in the analysis of mobility effect.

DRM-PV

One way forward is to preserve the diagonal-referencing principle of the DRM when constructing proxy variables for mobility — an approach we term DRM-PV. Specifically, DRM-PV takes the following form:

$$Y_{ijk} = \beta_0 + \beta_1 O_{ijk} + \beta_2 D_{ijk} + \beta_3\left[\frac{\widehat{K}_{ijk}(\pi_{ii}) + \widehat{K}_{ijk}(\pi_{jj})}{2}\right] + \varepsilon_{ijk}, \tag{3}$$

The right-hand side begins with the same terms as in Model (1), but rather than using a simple arithmetic difference between origin and destination identifiers, we employ a richer, more informative measure of mobility, $\frac{\widehat{K}_{ijk}(\pi_{ii}) + \widehat{K}_{ijk}(\pi_{jj})}{2}$. In this measure, π_{ii} denotes the profile of the diagonal cell (i,i) and π_{jj} carries the same meaning for the diagonal cell (j,j). The function $K_{ijk}(\pi_{ii})$ calculates the distance between observation k and π_{ii}; likewise, $K_{ijk}(\pi_{jj})$ measures the distance to π_{jj}. Thus, $\frac{\widehat{K}_{ijk}(\pi_{ii}) + \widehat{K}_{ijk}(\pi_{jj})}{2}$ represents the average distance of observation k in cell (i,j) to the two corresponding diagonal cells (i,i) and (j,j). As with DRM, this individual-level mobility measure, by referencing both diagonal cells, does not linearly vary with the combination of origin and destination and thus does not suffer from the identification problem. In this case, DRM-PV adopts the identification strategies of both proxy variable and nonlinearity. The nonlinearity strategy is necessary to circumvent the conceptual concerns raised in the previous section. This would distinguish DRM-PV from DRM.

The relationship between different modeling strategies is illustrated in Figure 1. The issue of non-identifiability emerges when the mobility term, $\beta_3(O_{ijk} - D_{ijk})$, is considered alongside the immobility term, $\beta_1 O_{ijk} + \beta_2 D_{ijk}$. To resolve this problem, one of these two terms must be modified. Specifically, DRM addresses the issue by replacing $\beta_1 O_{ijk} + \beta_2 D_{ijk}$ with the term $w \times \mu_{ii} + (1-w) \times \mu_{jj}$, whereas DRM-PV tackles it by substituting $\beta_3(O_{ijk} - D_{ijk})$ with

$$\beta_3\left[\frac{\widehat{K}_{ijk}(\pi_{ii}) + \widehat{K}_{ijk}(\pi_{jj})}{2}\right]$$

.

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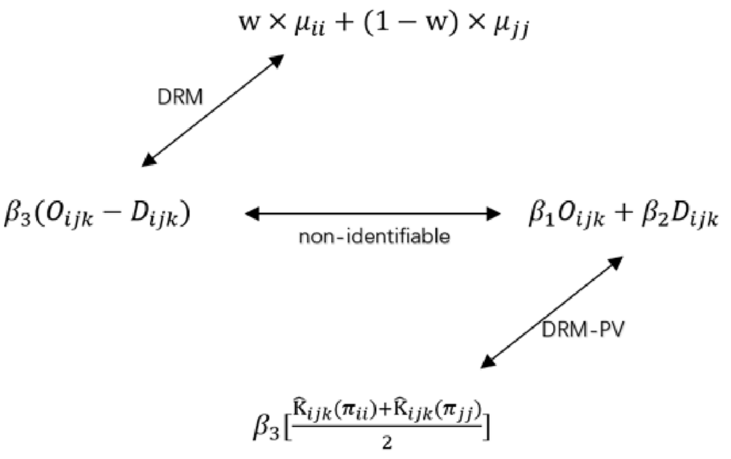
Figure 1

Comparison Between DRM-PV and DRM



With the model configuration of DRM-PV, the next question is how to practically measure the distance from a specific observation to the diagonal cells. Without loss of generality, we simplify the function $\widehat{K}_{ijk}$ to be the absolute value of the arithmetic difference. This allows us to express the measure of mobility as:

$$Mobility_{ijk} = \frac{\widehat{K}_{ijk}(\pi_{ii}) + \widehat{K}_{ijk}(\pi_{jj})}{2} = \frac{|\widehat{\pi}_{ijk} - \widehat{\pi}_{ii}| + |\widehat{\pi}_{ijk} - \widehat{\pi}_{jj}|}{2}, \tag{4}$$

where $\widehat{\pi}_{ijk}$ represents the characteristics of observation k in cell (i, j), and $\widehat{\pi}_{ii}$ and $\widehat{\pi}_{jj}$ refer to the characteristics of the diagonal cells (i,i) and (j,j), respectively. Inspired by the three proxy variables listed earlier, we also have three possible metrics to use, as shown in Table 1.

The first metric is $\frac{|\widehat{f}_{ij.} - \widehat{f}_{ii.}| + |\widehat{f}_{ij.} - \widehat{f}_{jj.}|}{2}$, where f refers to the empirical frequency or probability pass. By this metric, the distance is measured by the absolute value of the difference in empirical frequencies or probability masses between cell (i,j) and the diagonal cell (i,i). Note that individuals of the same cell in the mobility table would have the same value. That is why this metric does not carry a subscript k. As noted earlier, the substantive reason to adopt this metric is that the empirical frequency or probability mass represents the extent of permeability, in the sense that $\widehat{f}_{ij.}$ reflects how easily the mobility from i to j can be observed. Thus, $|\widehat{f}_{ij.} - \widehat{f}_{ii.}|$ is then the *relative ease* of realizing the mobility from status i to status j compared to remaining in status i. The same interpretation applies for $\left|\hat{f}_{ij.} - \hat{f}_{ii.}\right|$, so $\frac{|\hat{f}_{ij.} - \hat{f}_{ii.}| + |\hat{f}_{ij.} - \hat{f}_{jj.}|}{2}$ reflects the extent of permeability of reaching the cell i-j compared with staying at the nonmobile state.

The second approach is based on the predicted probabilities of being located in particular cells based on a set of individual-level covariates $\boldsymbol{X}$. For example, suppose there are M different cells ($M = I \times J$, where I is the number of rows and J is the number of columns of a mobility table), we may fit a multinomial logistic regression model to estimate the log-odds of being located in cell m relative to a reference cell, as in:

$$log\frac{P(M = m)}{P(M = reference)} = X\beta_m,$$

where m = 1,2, 3, ..., M-1, and β_m is the vector of coefficient. We use the multinominal logistic regression model because the outcome variable for this model is the cell identities of the mobility table, which represent nominal categories without an inherent ordering.

Based on this model, we can predict each individual's probability of being located in each cell of the mobility table. For example, for a 3×3 mobility table, each individual would obtain 9 predicted probabilities (the summation of them should be unity), including $\hat{p}_{ijk}$, the predicted probability of being in cell (i,j), and $\hat{p}_{iik}$, the predicted probability of being in the diagonal cell (i,i). Their difference $\left|\hat{p}_{ijk} - \hat{p}_{iik}\right|$ can then be used as a metric to gauge one's distance to the diagonal cell (i,i) in the atypicality sense. This is also the case for $\left|\hat{p}_{ijk} - \hat{p}_{jjk}\right|$, so the overall atypicality-based proxy with reference to the diagonal should be

$$\frac{\left|\hat{p}_{ijk} - \hat{p}_{iik}\right| + \left|\hat{p}_{ijk} - \hat{p}_{jjk}\right|}{2}$$

.

The final approach would be based on the idea of social distance, that is, to directly assess the distance of observation k in cell (i,j) to the diagonal cells based on covariates $\boldsymbol{X}$. Specifically, we compute the average value of $\boldsymbol{X}$ for cell (i,i), denoted as $\bar{x}_{ii.}$, and then calculate its discrepancy with the covariate values of observation k, x_{ijk}, that is, $\left|x_{ijk} - \bar{x}_{ii.}\right|$. This discrepancy can be computed using various metrics such as the Kullback–Leibler divergence. However, for mixed covariates — including both continuous and categorical variables — a more suitable choice is the Gower distance, which is simply the summation of absolute differences $\sum_{dk}\left|x_{ijk} - \bar{x}_{ii.}\right|$ (Gower, 1985). By combining the two diagonal cells, we have the proxy

$$\frac{\sum_{dk}\left|x_{ijk} - \bar{x}_{ii.}\right| + \sum_{dk}\left|x_{ijk} - \bar{x}_{jj.}\right|}{2}$$

.

Methodological Advantages of DRM-PV

Moving from DRM to DRM-PV shifts the focus from a reconstructed immobility term to the reconstruction of the mobility term. However, this is not merely a matter of looking at the other side of the same coin. On the contrary, DRM-PV provides empirical researchers with several methodological advantages.

First, DRM-PV directly estimates individual effects for all three variables:

- Origin
- Destination
- Mobility

Hence, the estimated mobility effect is a true "net" effect, as the individual measures of both origin and destination are controlled for when explaining a particular outcome.

Second, except for the frequency- or probability-mass- based metric, both the predicted probabilities and Gower distance utilize individual-level covariates, allowing for more nuanced analyses. For example, when comparing an observation to the average characteristics of the corresponding diagonal cell, cases may be matched to ensure comparable sociodemographic backgrounds (e.g., the same sex or race). If the focal respondent is female, a matching-based approach can replace the conventional diagonal reference — typically computed from all cases in the diagonal cell — with the average computed exclusively from female cases in that cell. This strategy has the advantage of purging the estimated mobility effect of potential confounding arising from sex differentials.

Third, diagonal cells are typically adopted as the reference category because they are presumed to capture the essential and stable characteristics of the nonmobile. In certain research settings, however, scholars may wish to invoke alternative reference points to interrogate different empirical contrasts. For example, one might benchmark all mobility cells against a "worst-case" category — such as individuals who experience downward mobility from the highest to the lowest socioeconomic status — in order to foreground asymmetries in mobility consequences. DRM-PV well accommodates this kind of reference shift. It is worth mentioning that this advantage is especially relevant when the substantive meaning of diagonal cells becomes ambiguous or hard to define, as in the case of large-scale structural mobility or income mobility.

Fourth, in research involving multiple mobility tables, DRM-PV enables counterfactual decomposition. Suppose we have two cohort groups, c1 and c2. We can estimate the counterfactual mobility effect for group c2, where the off-diagonal cells are characterized based on covariate values from c2, while the diagonal cells remain using covariate values from c1. This allows us to explore how inter-cohort changes in the characteristics of the non-mobile influence the mobility effect.

Fifth, DRM-PV can be extended to analyze income mobility. For example, in studying intergenerational income mobility, one can estimate the bivariate probability density between family income and individual income. This estimated density can then be used to construct the permeability and atypicality proxies (i.e., treating the estimated density to be a kind of frequency distribution, or to predict the point mass of the density based on covariates **X**).

Finally, the enriched measure of mobility — regardless of the specific metric adopted — offers an *empirically adaptive* alternative to the conventional measure that is based solely on difference in social status indicators. In the conventional intergenerational educational mobility analysis, for example, the mobility from elementary school to college is *ex ante* assumed to be a greater leap than that from high school to college, even without checking the empirical data. However, in contexts when access to college becomes almost universal regardless of parental background, these two routes of educational mobility may become equally attainable. Such an empirical mobility pattern cannot be captured by the conventional measure, but well accommodated and respected by DRM-PV, since the comparable routes of educational mobility are empirically manifestable by the close frequencies or predicted probabilities between the elementary-school-to-college route and high-school-to-college route.

Beyond these advantages, a direct comparison between DRM-PV and DRM yields additional insights. In DRM, mobility is modeled as a *residual* deviation from immobility net of diagonal references, implicitly treating all mobility experiences in a homogenous fashion. By contrast, DRM-PV represents mobility through theoretically motivated proxy variables (e.g., permeability, atypicality, and social distance), allowing researchers to identify which aspects of mobility are most consequential for the outcome — an interpretive refinement that DRM cannot offer. Whereas the mobility effect in DRM is defined implicitly by constraints on origin and destination effects and is therefore difficult to interpret, DRM-PV preserves the diagonal reference structure while redefining mobility in terms of observable, theory-driven proxies. This enables mobility effects to be *interpreted* more directly as operating through specific social processes. Moreover, because the proxy-based mobility terms in DRM-PV do not mechanically depend on the diagonal reference weights, DRM-PV provides a diagnostic perspective on the constraints underlying DRM, making it possible to assess whether the mobility effect persists once these constraints are relaxed, and to distinguish substantively meaningful mobility processes from effects driven primarily by model specification.

Illustrative Examples

Samples

We present two case studies to illustrate the applications of DRM-PV. One employs the conventional discrete measure of educational attainment, and the other uses a continuous measure of income to assess socioeconomic status. In the first example, we investigate the effect of intergenerational educational mobility on fertility using data from the

General Social Survey (GSS, for more details, see https://gss.norc.org/), spanning the years 1972 to 2022. The outcome variable is the total number of children ever born to the respondent. Educational attainment is measured as a three-category variable:

- 1 = less than high school.
- 2 = high school graduate.
- 3 = college degree.

This variable is recorded for both the respondents and their fathers. Following previous research, we use father's educational attainment due to its relative stability across historical periods (Luo, 2022).

The covariates used to predict the probabilities of being in each cell and to calculate the Gower distance are primarily related to individuals' values, judgments, or cultural characteristics. This selection of variables aligns with existing literature on the interplay between social and cultural mobility (Sorokin, 1959). In GSS, this kind of potential covariates can be as many as 67. However, many of them have extremely high rates of missing data — often between 60% and 70% — largely because they were not included in every ballot. To ensure data quality, we retain only covariates with less than 20% missing values, resulting in the following set of covariates:

- *Party identification* (1 = strong Democrat; 2 = not very strong Democrat; 3 = independent, close to Democrat; 4 = Independent (neither, no response); 5 = independent, close to Republican; 6 = not very strong Republican; 7 = strong Republican; 8 = other party).
- *Frequency of attending religious services* (1 = never; 2 = less than once a year; 3 = about once or twice a year; 4 = several times a year; 5 = about once a month; 6 = 2–3 times a month; 7 = nearly every week; 8 = every week; 9 = several times a week).
- *Subjective class identification* (1 = lower class; 2 = working class; 3 = middle class; 4 = upper class; 5 = no class).
- *Satisfaction with financial situation* (1 = pretty well satisfied; 2 = more or less satisfied; 3 = not satisfied at all).
- *Change in financial situation* (1 = better; 2 = worse; 3 = stayed same).
- *Opinion of family income* (1 = far below average; 2 = below average; 3 = average; 4 = above average; 5 = far above average).
- *General happiness* (1 = very happy; 2 = pretty happy; 3 = not too happy).
- *Political views* (1 = extremely liberal; 2 = liberal; 3 = slightly liberal; 4 = moderate, middle of the road; 5 = slightly conservative; 6 = conservative; 7 = extremely conservative).
- *Favor or oppose death penalty for murder* (1 = favor; 2 = oppose).

The results of the multinominal logistic regression model can be found in Figure A1 of the Supplementary Materials at Hu (2026). No additional sample restrictions are applied, yielding a total sample size of 54,265.

The second example uses data from the National Longitudinal Survey of Youth 1979 (NLSY79, for more details, see https://www.bls.gov/nls/nlsy79.htm) to examine the relationship between intergenerational income mobility and mental health status. We restrict the analysis to the period from 1978 up to the year the respondent turned 50. This upper bound is set because the mental health question was administered when respondents were around that age. We exclude individuals who were older than 19 in 1979 to avoid overrepresentation of late home-leavers (Bloome, 2014). The outcome variable is the Mental Health Component (MHC) score from the SF-12 Health Scale, using its original scale. Family income includes labor market earnings, assets, and both government and nongovernment transfers. It is adjusted for inflation using the core Consumer Price Index (CPI) Retroactive Series with 2018 as the reference year. To reduce measurement error, we average family income over the period 1978–1982 (survey years 1979–1983) and adjust for family needs by dividing it by the square root of family size (Bloome et al., 2018). Respondents' own income is also CPI-adjusted and averaged over ages 30 to 50. The lower age limit of 30 is used to reflect individuals' permanent income (Chetty et al., 2014; Haider & Solon, 2006). Both family and individual income variables are log-transformed. The final sample size is 4,068.

Empirical Results

We first fit four DRMs using a single weight w and one of four mobility variables: a dummy variable indicating *mobility status* (0 = non-mobile, 1 = mobile); a categorical variable capturing the *direction of social mobility* (0 = non-mobile, -1 = downwardly mobile, 1 = upwardly mobile); a variable indicating *the number of steps* an individual moves along the educational ladder (0, 1, or 2 steps); and a fourth variable that *combines mobility steps with mobility direction* (i.e., the product of direction and mobility steps). The results are presented in Table 2.

After controlling for the weighted sum of estimated diagonal cell means, Model A and Model B show significantly positive effects of mobility status and mobility direction, respectively, both at the 0.001 level. Model C, by contrast, shows a negative effect of the absolute number of mobility steps. However, this measure is of limited interpretative value, as it conflates the intensity of both upward and downward mobility. To address this concern, Model D combines mobility steps with direction — assigning negative values to downward mobility and positive ones to upward mobility. The resulting coefficient for mobility is significantly positive, indicating that individuals who experienced stronger upward mobility tend to have more children.

Table 2

Results of DRM

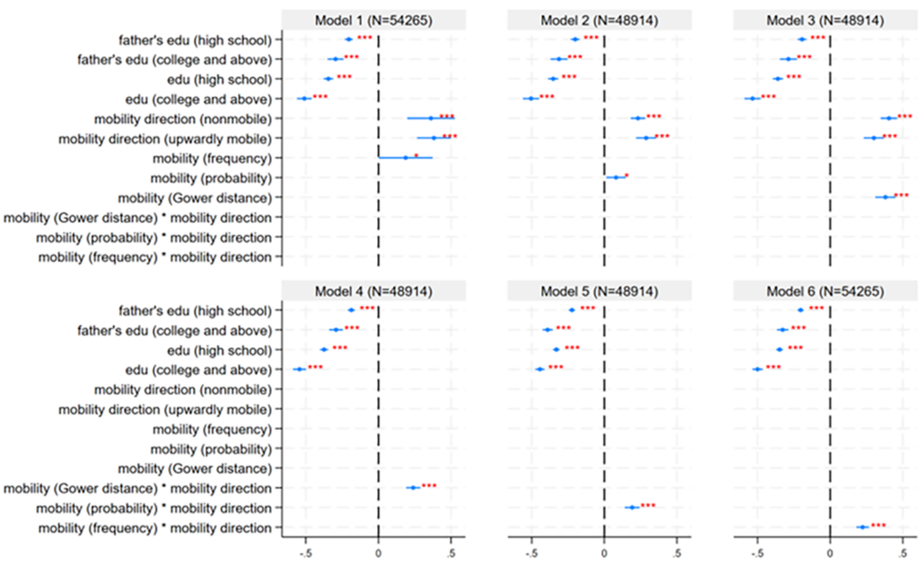
Variables (Diagonal cell means)	**Model A**	**Model B**	**Model C**	**Model D**
Below high school	0.892(0.014) ***	0.915(0.014) ***	0.915(0.014) ***	0.920(0.014) ***
High school	-0.238(0.013) ***	-0.255(0.012) ***	-0.285(0.014) ***	-0.242(0.013) ***
College and above	-0.653(0.014) ***	-0.661(0.013) ***	-0.631(0.014) ***	-0.677(0.014) ***
Weight (w)	0.561(0.014) ***	0.361(0.024) ***	0.439(0.026) ***	0.363(0.033) ***
Mobility (reference = nonmobile)	0.125(0.017) ***	0.371(0.031) ***	0.127(0.052)*	
Upward mobility		0.468(0.049) ***	0.358(0.048)***	
Absolute value of mobility steps			-0.183(0.033) ***	
Mobility steps combined with mobility direction				0.128(0.029) ***
Intercept	1.792(0.014) ***	1.548(0.029) ***	1.789(0.052) ***	1.872(0.009) ***
log likelihood	-104344.730	-104297.790	-104283.67	-104359.460
N	54265	54265	54265	54265

Note. Unstandardized coefficients with standard errors in parentheses. Data Source: General Social Survey 1972–2022.
*0.01 < p < 0.05. **0.001 < p < 0.01. *** p < 0.001 (two-tailed test).

Figure 2 displays the results of DRM-PV (detailed results are shown in Table A1 of the Supplementary Materials at Hu, 2026). Model 1 considers both mobility direction and the permeability-based measure. All three predictors — direction, frequency, and the mobility metric — show statistically significant coefficients. Similar findings are observed in Model 2 (based on atypicality) and Model 3 (based on social distance). Models 4 through 6 combine mobility direction with each of the three enriched mobility measures. The results remain consistent with those of DRM: A significantly positive association between mobility and fertility, after accounting for origin and destination effects. Note that one of the strengths of DRM-PV is its capacity to estimate both origin and destination effects directly. Regardless of model specification, increases in educational attainment — for both the father and the respondent — are associated with lower fertility. This is not uncovered by the traditional DRM. In addition, alternative proxy specifications yield substantively consistent results. This convergence suggests that distinct dimensions of mobility exert similar effects on fertility, thereby reinforcing the robustness of the conclusions.

Figure 2

Results of Different Configurations of DRM-PV



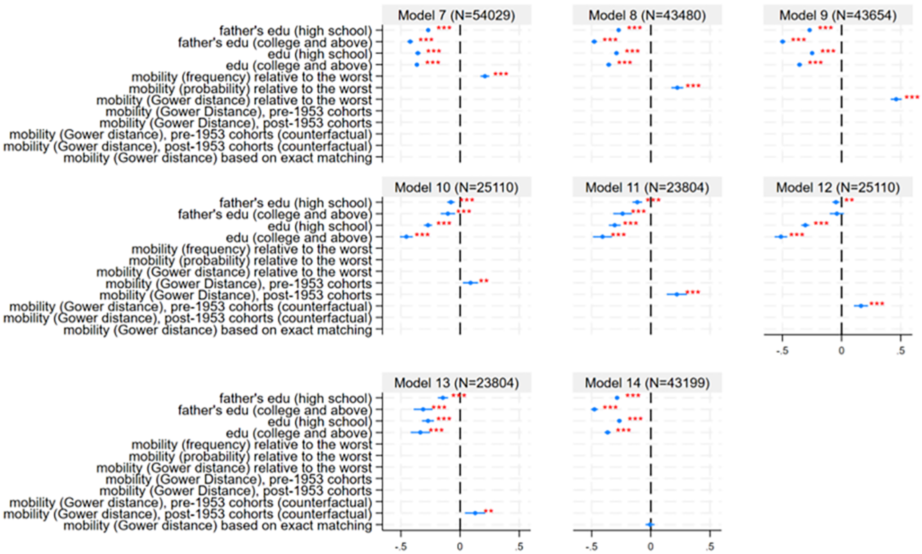
Note. Point estimates and their 95% confidence intervals. Mobility direction and mobility are examined separately for Models 1 through 3, while the interaction between mobility measure and mobility direction is used for Models 4 through 6. All models include father's educational attainment and respondent's educational attainment. No estimate is marked out if a variable is not considered by a particular Model. Data Source: General Social Survey 1972–2022.

*0.01 < p < 0.05. **0.001 < p < 0.01. ***p < 0.001 (two-tailed test).

Figure 3 presents additional applications of DRM-PV (detailed results are shown in the Supplementary Materials in Table A2 of Hu, 2026). The first set of models (Models 7 through 9) shifts the reference group from the diagonal to the "worst-case" — individuals who attained less than a high school education while their fathers held a college degree or higher. Compared to the results based on diagonal reference, the estimated mobility effect using Gower distance is notably stronger, with the point estimate increasing from 0.223 to 0.461 ($p < 0.001$, two-tailed test).

Figure 3

Extended Analyses Using DRM-PV



Note. Point estimates and their 95% confidence intervals. All models include father's educational attainment and respondent's educational attainment. No estimate is marked out if a variable is not considered by a particular Model. Data Source: General Social Survey 1972–2022.
*0.01 < p < 0.05. **0.001 < p < 0.01. ***p < 0.001 (two-tailed test).

Next, we divide the sample into two subgroups: those born before 1953 and those born after. We use 1953 as the dividing line because it represents the median birth year in the GSS. DRM-PV results for these subgroups (Model 10 and Model 11) reveal that the mobility effect is significantly stronger for the younger cohort (0.089 vs. 0.220, $p < 0.01$, two-tailed test). We then conduct a counterfactual analysis by fitting DRM-PV for the pre-1953 group using the diagonal cell information from the post-1953 group (Model 12). This substitution increases the mobility effect on fertility from 0.089 to 0.164, marginally significant ($p = 0.076$, two-tailed test). A similar counterfactual analysis using the pre-1953 diagonal cell information for the post-1953 group (Model 13) leads to a decrease in the mobility effect from 0.220 to 0.128 (p = 0.10, two-tailed test). These results suggest that cohort-based changes in the non-mobile (diagonal) contribute to the observed variation in the mobility effect on fertility.

Model 14 explores the mobility effect based on exact matching on sex and race. As previously described, this procedure restricts comparisons with cases of the same gender *and* race. The resulting model shows a non-significant mobility effect, implying that some portion of the previously observed mobility effect may actually arise from cross-sex or cross-race variations.

DRM-PV can be also used for examining how income mobility relates to mental health outcomes (detailed results are shown in the Supplementary Materials, Table A3 at Hu, 2026). Using the NLSY79, Figure 4(a) presents a nonparametric density heatmap for bivariate distribution between log family income and log adult income, where most estimated density values are close to zero, with larger values concentrated in the top-right corner. We then adopt the frequency-based approach to construct the variable of mobility. Unlike categorical educational attainment, income mobility lacks clearly defined diagonal "cells," making conventional diagonal reference inapplicable. One practical alternative is to compare the estimated density at each individual's observed income coordinates to the density corresponding to the *median values* of both family and adult income. However, because the density at the medians is a constant, this referencing to the median probability density is tantamount to subtracting a constant from all estimated densities — an adjustment that can be absorbed by the intercept in a linear regression model. Thus, the estimated density values themselves can be used directly as a continuous indicator of mobility. Figure 4(b) presents the results of this analysis. Regardless of whether the density metric is used alone (Model 15) or in combination with the mobility direction indicator (Model 16), we find no significant mobility effect on mental health outcomes.

Figure 4a

Results of the Income Mobility Effect: Estimates of Bivariate Density

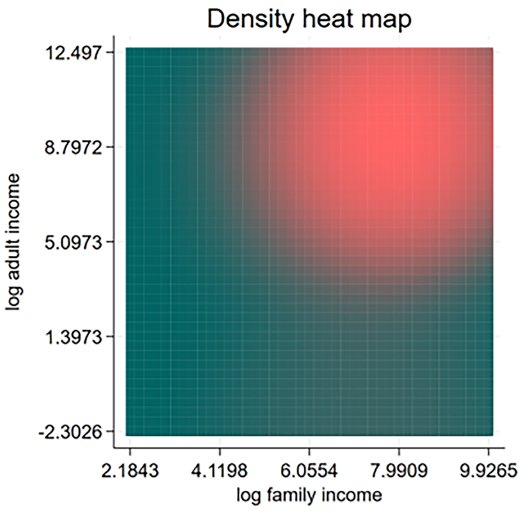
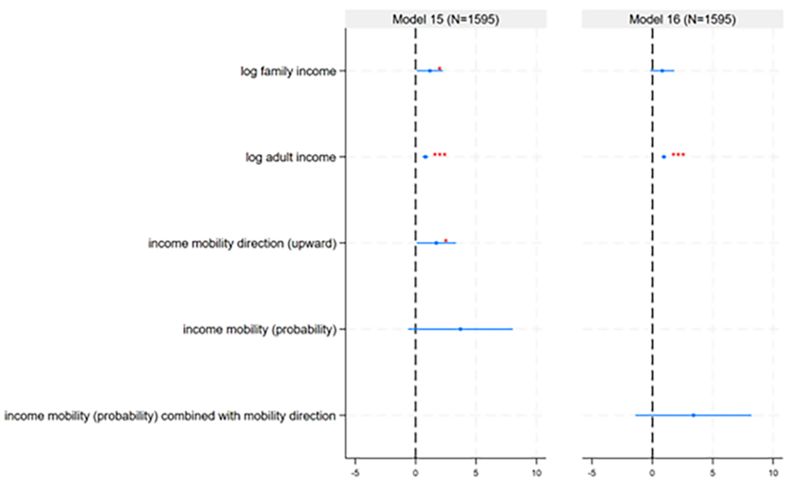


Figure 4b

Results of the Income Mobility Effect: Results of DRM-PV



Note. Point estimates and their 95% confidence intervals for (b). Both models include log family income and log adult income. No estimate is marked out if a variable is not considered by a particular Model. Data Source: The National Longitudinal Survey of Youth 1979.

*0.01 < p < 0.05. **0.001 < p < 0.01. ***p < 0.001 (two-tailed test).

Concluding Remarks

To address the identification problem in estimating the mobility effect, a variety of strategies and modeling approaches have been proposed. Most existing models — including the prevailing DRM — ultimately rely on a nonlinearity-based solution, often through explicit or implicit constraints whose substantive plausibility has been questioned. In this study, we shift attention to the proxy-variable approach. Rather than treating mobility as a nominal difference between socioeconomic status indicators, this approach interrogates what substantively underlies that difference by enriching the mobility term with theoretically meaningful and empirically informative proxies. Specifically, we propose three proxies corresponding to the permeability, atypicality, and social-distance dimensions of the mobility process. Under this formulation, mobility is no longer represented as a simple difference between class indicators. Instead, it explicitly reflects variation in empirical likelihoods of observation, disparities in predicted probabilities, or covariate-based distances derived from multiple dimensions.

Regardless of the metric employed, this enriched conceptualization adds substantive depth to an otherwise simplistic identifier difference. It not only enables the simultaneous estimation of the net effects of origin, destination, and mobility, but also opens up previously understudied research scenarios, including but not limited to reference-group shifts, counterfactual decomposition, sociodemographic matching, and analyses of income mobility. The empirical examples illustrate the use of DRM-PV with both categorical mobility tables and continuous measures of socioeconomic status, suggesting that DRM-PV serves as a flexible and practical analytical tool for empirical researchers.

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Data Availability: Both datasets are publicly available at ICPSR. Supplementary figures and tables are available at Hu (2026).

Supplementary Materials

Type of supplementary material	Availability/Access
Data	
No data provided	–
Code	
No code available	–
Material	
Supplementary figures and tables	Hu (2026)
Study/Analysis preregistration	
No preregistration	–
Other	
No other material has been made available	–

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